Higher Secondary School Main Exam - 2024

Subject – Maths

Class – XII

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. All the questions are compulsory.

2. Sub questions of Question Nos. 1 to 5 carry 1 mark each.

3. Question Nos. 6 to 15 carry 2 marks each.

4. Question Nos. 16 to 19 carry 3 marks each.

5. Question Nos. 20 to 23 carry 4 marks each.

1. Choose and write the correct options: **$1 \times 6 = 6$**

(i) In $f : R \to R$, be defined as $f(x) = x^2$ them

(A) f is one–one onto (B) f is many–one onto

(C) f is one–one but not onto (D) f is neither one–one nor onto

(ii) The principal value of $\sin^{-1}\left(-\frac{1}{2}\right)$ is

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{6}$

(C) $\frac{4\pi}{3}$ (D) $\frac{2\pi}{3}$

(iii) The number of all possible matrices of order 2×2 with entry 0 or 1 is

(A) 27 (B) 512

(C) 16 (D) 2

(iv) The probability of obtaining an even prime number on each die, when a pair of dice is rolled is

(A) 0 (B) $\frac{1}{3}$

(C) $\frac{1}{12}$ (D) $\frac{1}{36}$

(v) If A is a nonsingular square matrix of order and |A|=2 then |adj A| is equal to -

(A) 4 (B) 2

(C) 8 (D) 0

(vi) If $\vec{a}$ and $\vec{b}$ are two non-zero vectors, then $\vec{a}.\vec{b} = -|\vec{a}||\vec{b}|$ if

(A) $\theta = \frac{\pi}{2}$ (B) $\theta = 0°$

(C) $\theta = \pi$ (D) $\theta = \frac{3\pi}{2}$

2. Fill in the blanks : **1 × 7 = 7**

(i) A Relation R in a set A is said to be _____relation if R is reflexive, symmetric, and transitive.

(ii) The value of an inverse trigonometric functions which lies in its principal value branch is called _____ value of that inverse trigonometric functions.

(iii) If A'=A then A is called _____matrix.

(iv) value of $\begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix}$ is_____.

(v) Differenctial coefficient of e^{-x} is _____.

(vi) A point C in the domain of a function f at which either f'(C) = 0 or f is not differentiable is called a _____point of f.

3. Write true or false: **1 × 6 = 6**

(i) A function $f : X \to Y$ is one-one if $f(x_1) = f(x_2) \Rightarrow x_1 \neq x_2 \forall x_1, x_2 \in X$.

(ii) The range of principal value branch of $\cot^{-1}$ is $(0, \pi)$

(iii) Any square matrix can be represented as the sum of a symmetric and a skew symmetric matrix.

(iv) Multiplication of diagonal matrices of same order will be commutative.

(v) The differential coefficient of $\frac{1}{x}$ with respect to x is log x.

(vi) If l_1, m_1, n_1 and l_2, m_2, n_2 are the direction cosines of two lines and θ is the acute angle between the two lines, then $\sin\theta = |l_1l_2 + m_1m_2 + n_1n_2|$

4. Match the correct pairs:

	Column 'A'		**Column 'B'**
(i)	$\int \sqrt{x^2 - a^2}\, dx$	(a)	$\frac{1}{2a}\log\left\|\frac{x-a}{x+a}\right| + c$
(ii)	$\int \sqrt{a^2 - x^2}\, dx$	(b)	$\sin^{-1}\frac{x}{a} + c$
(iii)	$\int \frac{dx}{\sqrt{x^2 - a^2}}$	(c)	$\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + c$
(iv)	$\int \frac{dx}{\sqrt{a^2 - x^2}}$	(d)	$\log\left\|x + \sqrt{x^2 - a^2}\right| + c$
(v)	$\int \frac{dx}{x^2 - a^2}$	(e)	$\frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\log\left\|x + \sqrt{x^2 - a^2}\right| + c$
(vi)	$\int \frac{dx}{a^2 - x^2}$	(f)	$\sec^{-1}x$

(vii)	Simplest form of $\cot^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right), x>1$	(g)	$\frac{1}{2a}\log\left\|\frac{a+x}{a-x}\right\|+c$
		(h)	$\frac{1}{a}\tan^{-1}\frac{x}{a}+c$
		(g)	$\tan^{-1}x$

5. **Given answers in one word/sentence each:** **$1\times 7 = 7$**

(i) Write minimum value of the function f given by $f(x)=x^2$. $x\in R$.

(ii) If A and B are two events such that $A\subset B$ and $P(A)\neq 0$, then write value of $P\left(\frac{B}{A}\right)$

(iii) If $x\left(\hat{i}+\hat{j}+\hat{k}\right)$ is a unit vector, write the value of x

(iv) Write the value of $\hat{i}\left(\hat{j}\times k\right)+\hat{j}\left(\hat{i}\times k\right)+k\left(\hat{i}\times\hat{j}\right)$.

(v) Write the degree of differential equation $xy\frac{d^2y}{dx^2}+x\left(\frac{dy}{dx}\right)^2-y\frac{dy}{dx}=0$

(vi) Write the integrating factor of differential equation $\frac{dy}{dx}+2y=\sin x$.

(vii) Write the number of arbitrary constants in the particular solution of a differential equation of third order.

6. Show that a one-one function $f:\{1, 2, 3\}\to\{1, 2, 3\}$ must be onto.

2

Or

Show that the relation R in the set {1,2,3} given by {(1, 2), (2, 1), (2, 3), (3, 2)} is symmetric but neither reflexive nor transitive.

7. Prove that $\tan^{-1}\sqrt{x}=\frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right)$, $x\in[0,1]$ **2**

Or

Find the value of $\tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right]$.

8. Solve the given equation for x, y, z and t, if $2\begin{bmatrix}x & z\\ y & t\end{bmatrix}+3\begin{bmatrix}1 & -1\\ 0 & 2\end{bmatrix}=3\begin{bmatrix}3 & 5\\ 4 & 6\end{bmatrix}$.

2

Or

For the matrix $A=\begin{bmatrix}1 & 5\\ 6 & 7\end{bmatrix}$, verify that (A–A') is a skew symmetric matrix.

9. If 2x+3y=sinx, then find $\frac{dy}{dx}$. **2**

Or

If x=acos θ , y=asin θ , then find $\frac{dy}{dx}$.

10. Show that the function given by *f* (x) = 3x +17 is increasing on R. 2

Or

Prove that the function given by f(x)=cosx is decreasing in $(0, \pi)$.

11. The radius of a circle is increasing at the rate of 0.7 cm/s. What is the rate of increase of its circumference when r = 4.9 cm? **2**

Or

The radius of a circle is increasing uniformly at the rate of 3 cm/s. Find the rate at which the area of the circle is increasing when the radius is 20 cm.

12. Find the value of : $\int x\, e^x\, dx$ **2**

Or

Find the value of : $\int \frac{2-3\sin x}{\cos^2 x}\, dx$

13. Prove that $\left(\vec{a}+\vec{b}\right)\cdot\left(\vec{a}+\vec{b}\right)=|\vec{a}|^2+|\vec{b}|^2$ if and only if $\vec{a}$, $\vec{b}$ are perpendicular, given $\vec{a}\neq\vec{0}$, $\vec{b}\neq\vec{0}$. **2**

Or

If $\vec{a}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{b}=\hat{i}+2\hat{j}+3\hat{k}$ then find $\left(\vec{a}+\vec{b}\right)\times\left(\vec{a}-\vec{b}\right)$.

14. Find the angle between the vectors $\left(\hat{i}-2\hat{j}+3\hat{k}\right)$ and $\left(3\hat{i}-2\hat{j}+\hat{k}\right)$. **2**

Or

Find the projection of the vector on the $\hat{i}+3\hat{j}+7\hat{k}$ on the vector $7\hat{i}-\hat{j}+8\hat{k}$.

15. Find the equation of a line in vector form that passes through the point with position vector $2\hat{i}-\hat{j}+4\hat{k}$ and is in the direction $3\hat{i}+2\hat{j}-2\hat{k}$. **2**

Or

Show that the lines $\frac{x+5}{7}=\frac{y+2}{-5}=\frac{z}{-1}$ and $\frac{x}{-1}=\frac{y-1}{-2}=\frac{z+2}{3}$ are perpendicular to each other.

16. Using integration, find the area enclosed by the circle $x^2+y^2=a^2$. **3**

Or

Using integration, find the area enclosed by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

17. Find the equation of the curve passing through the point (1, 1) whose differential equation is $xdy=(2x^2+1)\,dx$, $(x \neq 0)$ **3**

Or

Find the general solution of the differential equation $\frac{dy}{dx}+\frac{y}{x}=x^2$.

18. Solve the following linear programming problem graphically : **3**

Maximise $Z = 4x + y$

Subject to the constraints :

$x+y \leq 50$

$3x+y \leq 90,\ x \geq 0,\ y \geq 0$

Or

Maximize $Z = 3x + 2y$ subject to the constraints :

$x + 3y \leq 60,\ x + y \geq 10,\ x \leq y,\ x \geq 0,\ y \geq 0$

19. Two bags 1 and 11 are given . Bag 1 contains 3 red and 4 black balls. While another bag 11 contains 5 red and 6 black balls. One is drawn at random from one of the bags and it is found to be red. Find the probability that it was drawn from Bag 11. **3**

Or

Three cards are drawn successively, without replacement from a pack of 52 well shuffled cards. What is the probability that first two cards are kings and the third card drawn is an ace?

20. If $A=\begin{bmatrix}2 & 3\\ 1 & -4\end{bmatrix}$, $B=\begin{bmatrix}1 & -2\\ -1 & 3\end{bmatrix}$, then verity that $(AB)^{-1}=B^{-1}A^{-1}$. **4**

Or

Solve, given system of linear equations. Using matrix method:

$5x + 2y = 4$

$7x + 3y = 5$

21. If $y=3\cos(\log x)+4\sin(\log x)$, show that $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=0$. **4**

Or

Find the relationship between a and b so that the function *f* defined by

$$f(x)=\begin{cases}ax+1 & \text{if } x \leq 3\\ bx+3 & \text{if } x>3\end{cases}$$

is continuous at x = 3

22. Evaluate

$\int_0^2 x\sqrt{2-1}\,dx$

Or

Evaluate

$\int x\tan^{-1}x\,dx$

23. Find the shortest distance between lines.

$\vec{r}=6\hat{i}+2\hat{j}+2k+\lambda\left(\hat{i}-2\hat{j}+2k\right)$ and $\vec{r}=-4\hat{i}-k+\mu\left(3\hat{i}-2\hat{j}-2k\right)$ **4**

Or

Find the value of P so that the lines

$\frac{1-x}{3}=\frac{7y-14}{2p}=\frac{z-3}{2}$ and $\frac{7-7x}{3p}=\frac{y-5}{1}=\frac{6-z}{5}$

are perpendicular.

***** End *****